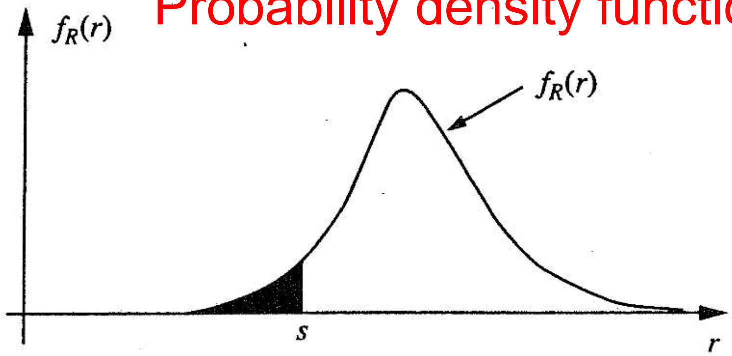
Structural Reliability Analysis – A Simple Case

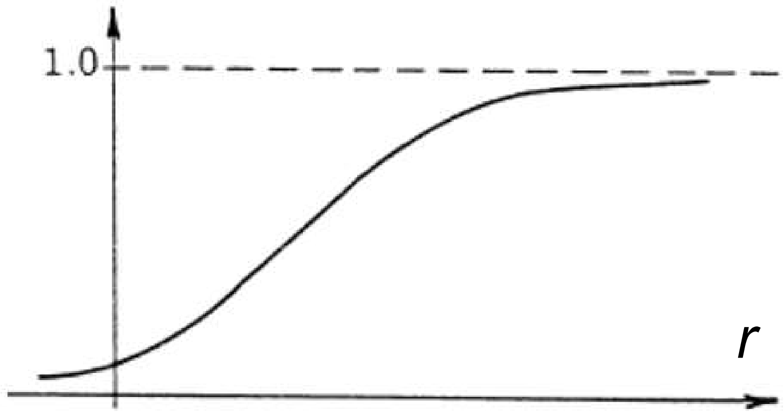
Bar with *random* strength subjected to a *known* axial load

Strength	➨	random variable R
Load	➨	deterministic value *s*

$F_R(r)$ Probability Distribution Function

Probability density function





Failure when R is smaller than *s*

$F_R(r) = P(R \le r)$

Probability of failure $= p_f$

$= \text{Prob}(R \le s)$

$= \text{Prob}(R - s \le 0)$

$= F_R(s)$

$$= \int_{-\infty}^{s} f_R(r)\, dr$$

Reliability $= 1 - p_f$

Structural Reliability Analysis – Example

Example 1: Involving one random variable – Simple Case

Consider a steel bar which is to be tested in the laboratory in a standard tensile rig. A fixed axial load is applied at one end, whilst the bar is rigidly held at the other end. We wish to estimate the probability that the bar will yield under a deterministic (accurately measured by a load cell) applied load of s kN. In the following, s will take different numerical values in order to illustrate how the probability against yield changes as s changes.

Assume that the yield stress of the material is a normally distributed random variable with a mean value of 420 MPa and a standard deviation of 25 MPa. Further, assume that the cross section of the bar is accurately measured and equal to 144 mm^2 (i.e. 12mm x 12mm).

Structural Reliability Analysis – Example

Example 1: Involving one random variable – Simple Case

Clearly, R = Yield Stress x Area = f_y x A *the resistance random variable*

where f_y = N (420, 25) {in MPa} *- a normally distributed random variable*
A = 144 mm^2 *a deterministic quntity, a constant*

Thus, R = N (60.48, 3.60) {in kN}
i.e. a normally distributed random variable with mean μ_R = 60.48 kN and standard deviation, σ_R = 3.60 kN.

Reminder: The above is obtained using E[aX] = a E[X] and Var[aX] = a^2 Var[X] where a is constant and X is a random variable (see expectation properties); moreover, a variable which is a linear function of normally distributed variables is also a normal variable.

Structural Reliability Analysis – Example

Example 1: Involving one random variable – Simple Case

In this case the load is deterministic, taking any particular value, say s. The probability of failure can then be written as (see Example 1)

$$p_f = \text{Prob}\,(R \leq s)$$

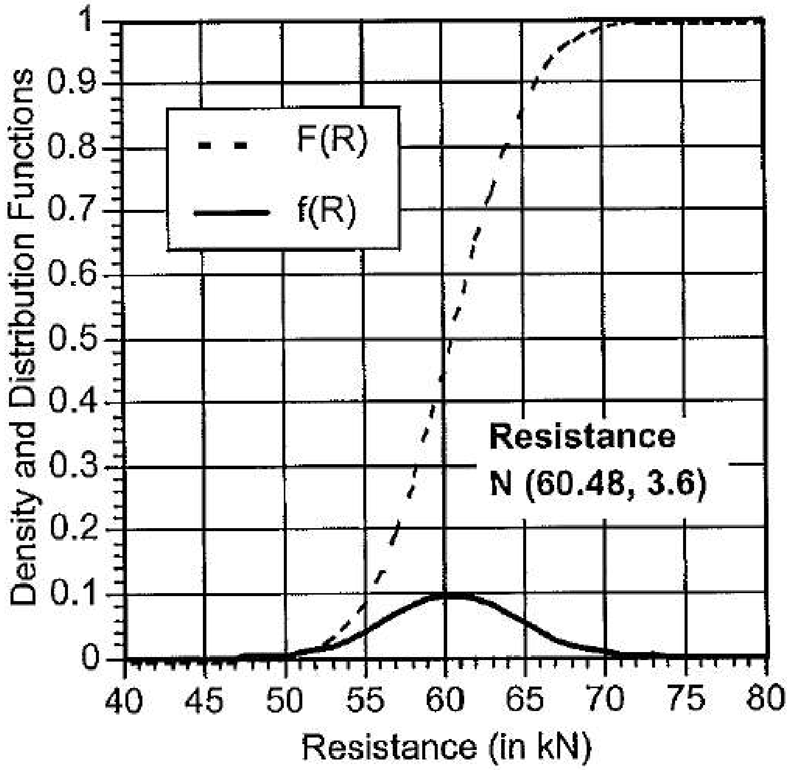
$$= \int_{-\infty}^{s} f_R(r)\,dr \;=\; F_R(s)$$

where $f_R(r)$ is the probability density function of R given by

$$f_R(r) = \frac{1}{\sigma_R\sqrt{2\pi}}\exp\left[-\frac{1}{2}\frac{(r-\mu_R)^2}{\sigma_R^2}\right] \qquad \text{for} \quad -\infty \leq r \leq \infty$$

with $\mu_R = 60.48$ kN and $\sigma_R = 3.60$ kN

and $F_R(s)$ is the distribution function of R evaluated at $R = s$.



Structural Reliability Analysis – Example

Example 1: Involving one random variable – Simple Case

As in Example 1, for this case, where there is only one random variable, the probability of failure is found by integrating the probability density of R from $-\infty$ to s; or by evaluating the cumulative distribution of R at s.

In this example, R is normal, hence in order to perform this calculation we need to transform R to a standard normal variable, which we will denote with Z_R.

Reminders:
(1) a standard normal variable is N (0,1), i.e. it has zero mean and unit standard deviation;
(2) any normal variable can become a standard normal through a linear transformation;
(3) for estimating probabilities involving a normal variable all we need is the standard normal table

$$R \rightarrow Z_R = \frac{R - \mu_R}{\sigma_R}$$

Thus,

$$\text{Prob}\,[R \leq s] = \text{Prob}\,[Z_R \leq \frac{s - \mu_R}{\sigma_R}] = \text{Prob}\,[Z_R \leq \frac{s - 60.48}{3.6}] = \Phi(\frac{s - 60.48}{3.6})$$

where $\Phi(.)$ is the standard normal distribution tabulated in textbooks.