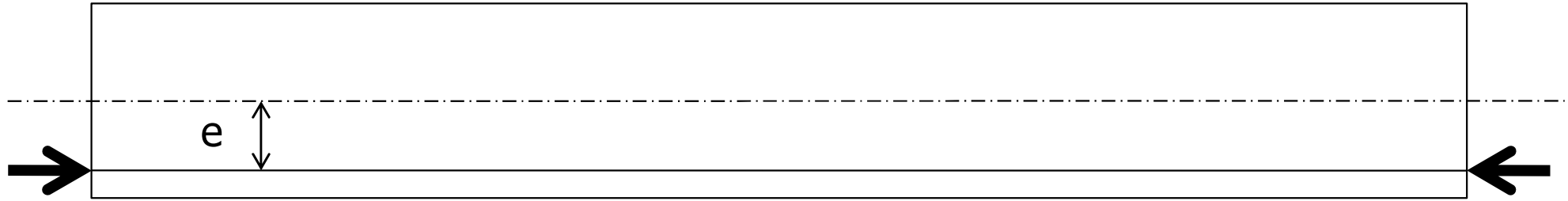
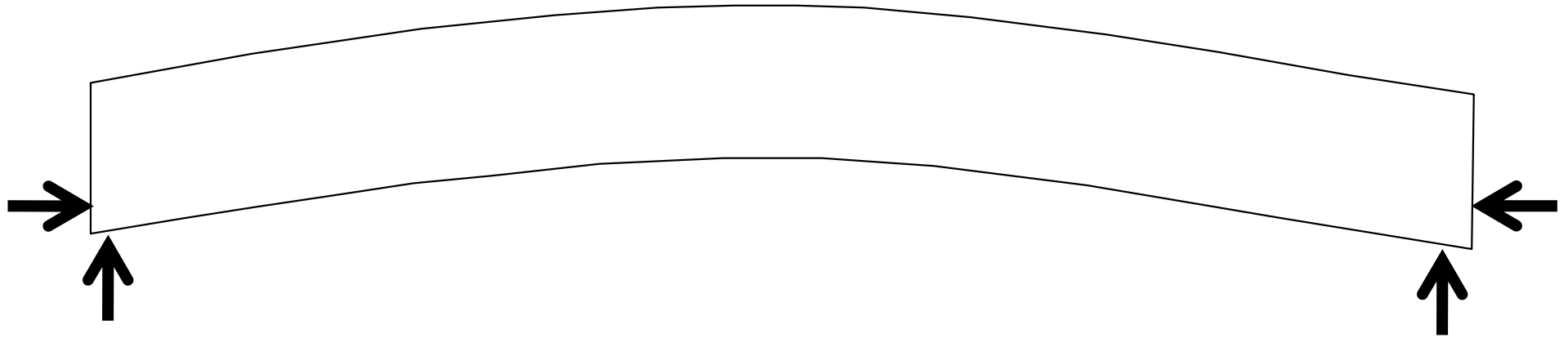


Secondary Effects



$$M = P \cdot e$$



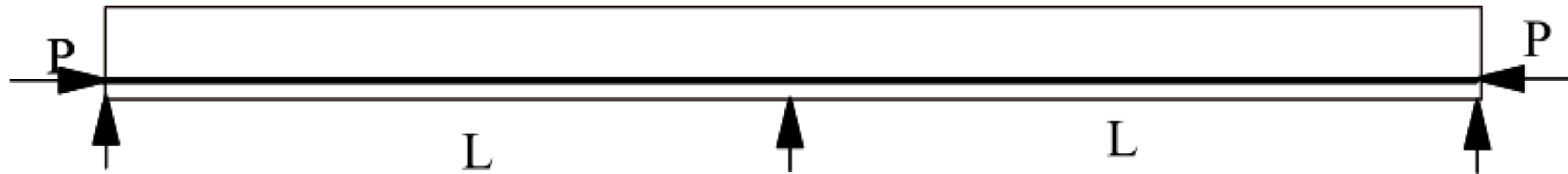
Simply supported beams have no restraints against translation and rotation (deformations).

Secondary Effects

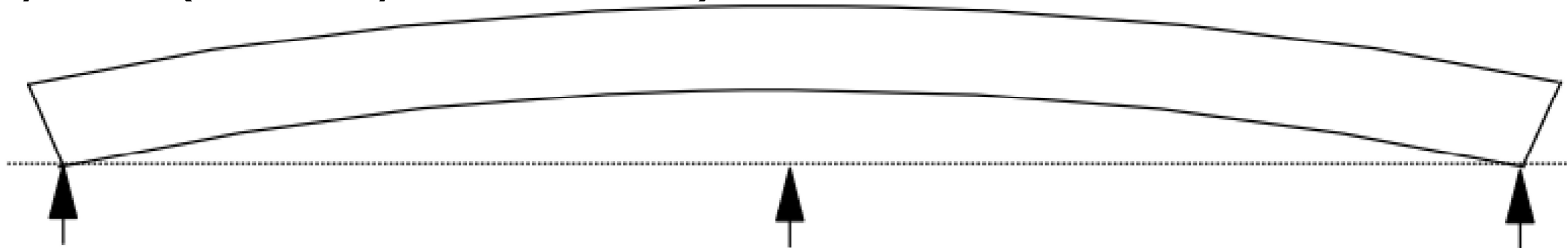
The magnitude of reaction, and hence that of the secondary effects depends upon:

- The magnitude of prestressing force
- The layout of the beam
- Tendon profile

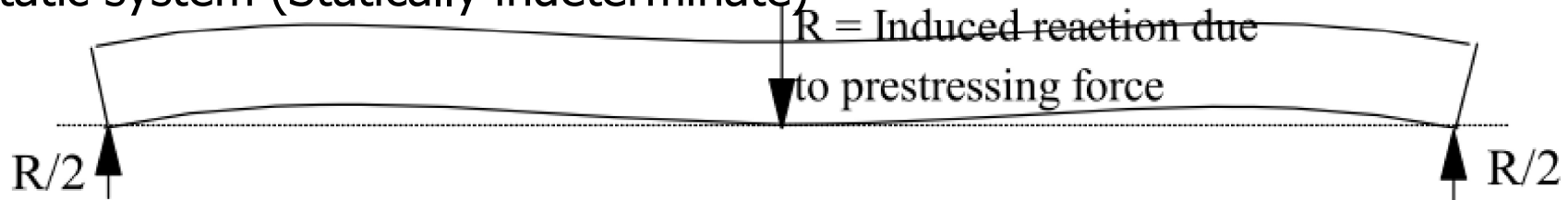
Secondary Effects



Isostatic system (Statically determinate)

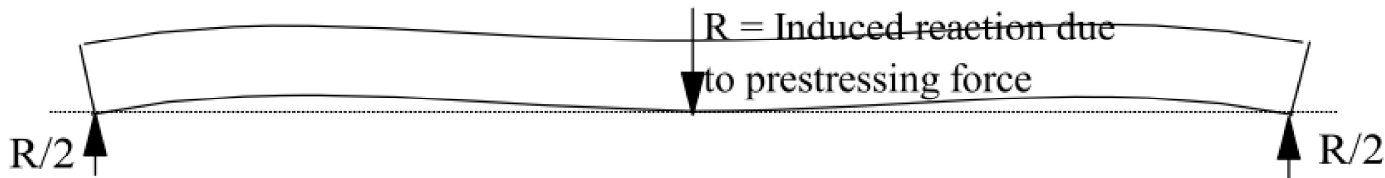
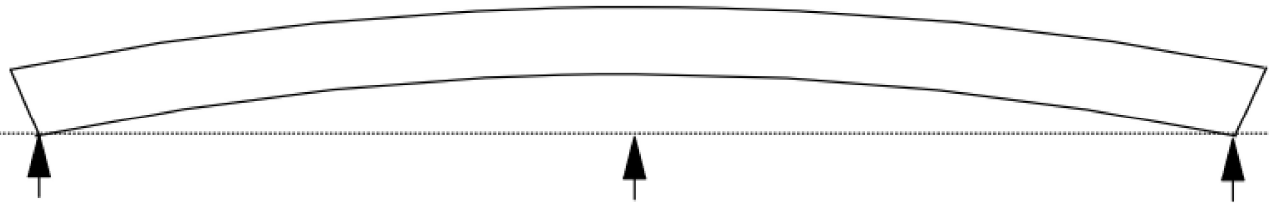
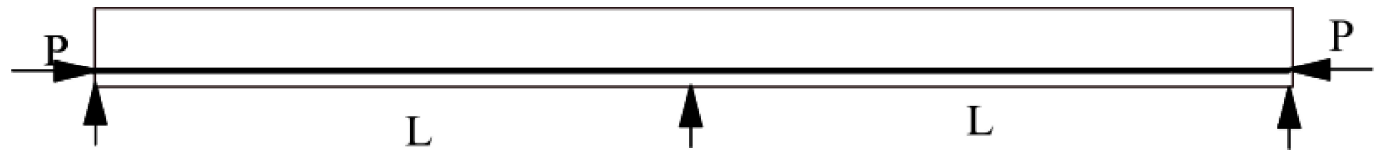


Hyperstatic system (Statically indeterminate)



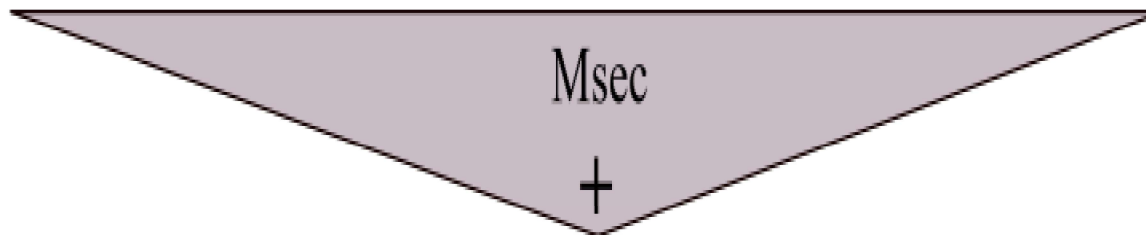
In continuous members, additional restraints at supports causes secondary effects.

Secondary Effects



P_e Primary BMD

statically determinate



Secondary BMD

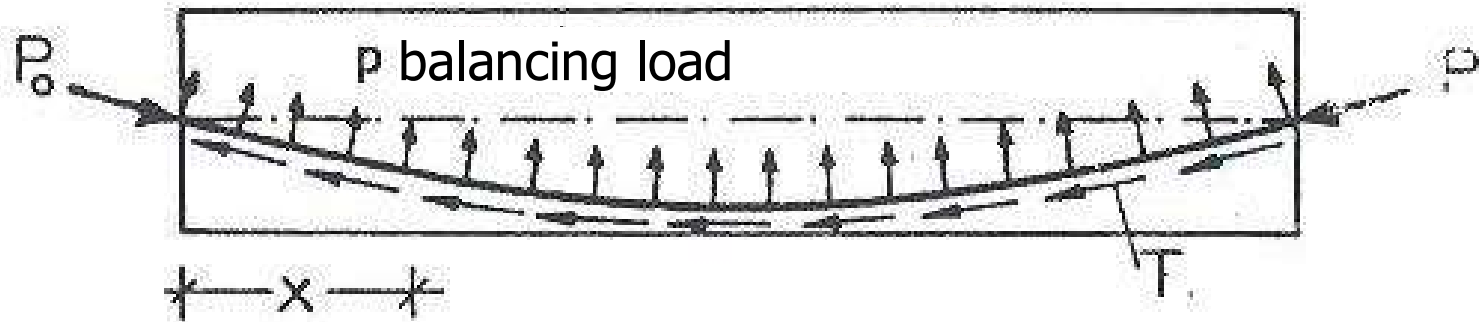
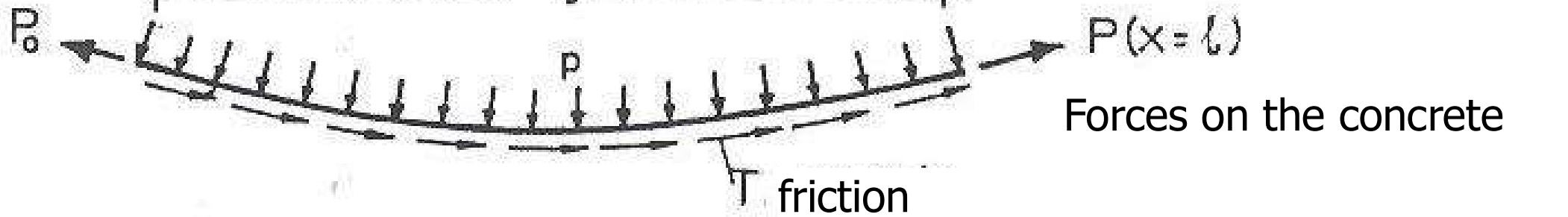
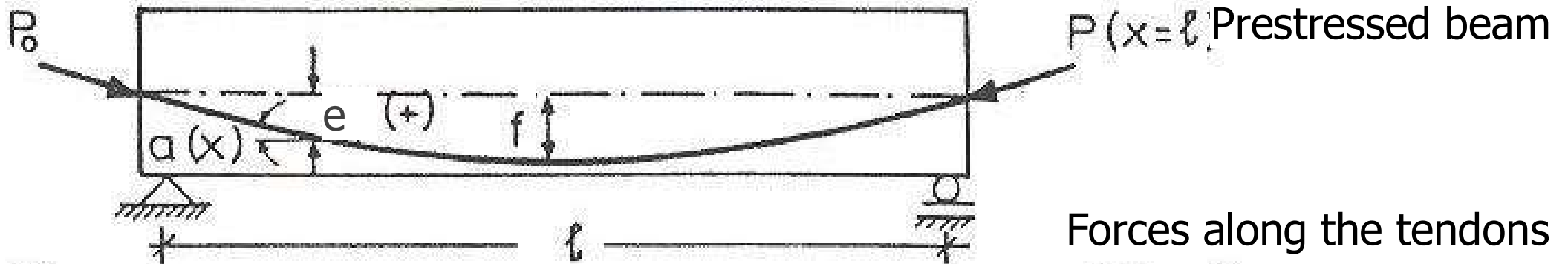
Guidance

Is the structural system statically indeterminate?

If **yes** then secondary moments are developed if:

- The tendon has **eccentricity** with respect the CG of the section (to a non-fixed end) or when
- The tendon has **curvature** or changes in direction (**polygonal** tendon)

For the above cases balancing load method can be used



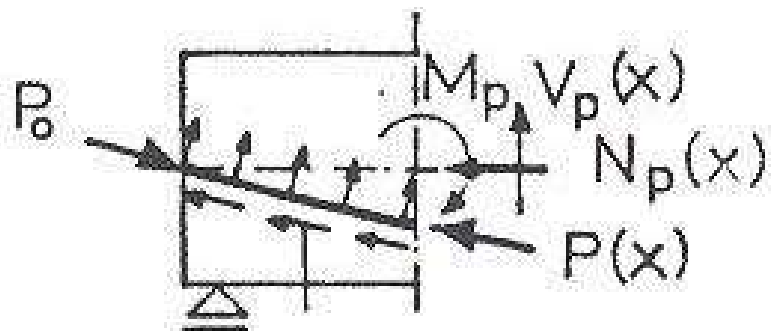
How much are the reactions here?

Internal actions-Equilibrium

$$N_p(x) = -P(x) \cdot \cos \alpha(x)$$

$$M_p(x) = -e(x) \cdot P(x) \cdot \cos \alpha(x)$$

$$V_p(x) = -P(x) \cdot \sin \alpha(x)$$



$$dV = \frac{p}{\cos da} dS \quad da = \frac{dS}{R}$$

$$dV = P \sin da \cong P da \quad \rightarrow$$

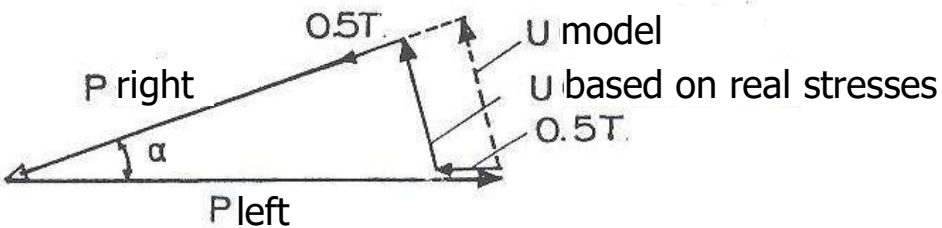
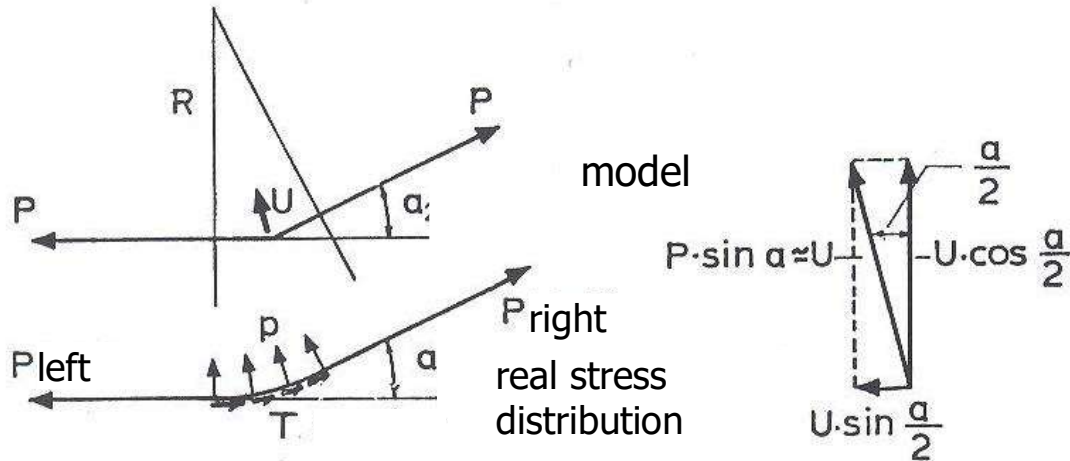
$$p = \frac{dV \cos da}{dS} = \frac{P \sin da \cos da}{dS} = \frac{P da}{dS} = P \cdot \frac{1}{R} = -uP$$

$$p = -uP$$

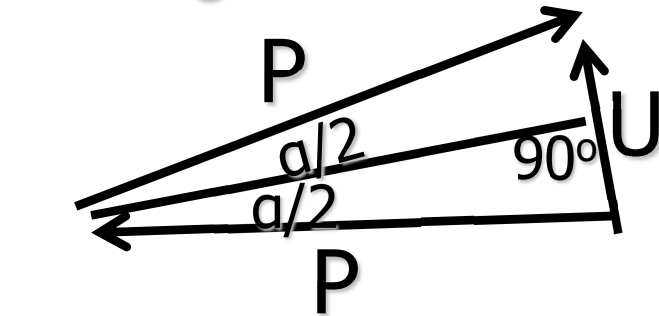
And the curvature can be calculated by the tendon's parabola:

$$u = (-1/R) = \frac{-8 f}{\ell^2}$$

Show that: $U = Pa$ when tendon changes direction



$$U = \begin{cases} P \cdot \sin \alpha & \text{model (without friction)} \\ \frac{1}{2} [P_{\text{left}} + P_{\text{right}}] \sin \alpha & \text{(with friction)} \end{cases}$$



From the above figure:

$$U/2 = P \cdot \sin(\alpha/2)$$

$$U = 2 P \cdot \sin(\alpha/2) = P \cdot \alpha$$

Remember : $\sin(\alpha) = 2 \cdot \sin(\alpha/2) \cdot \cos(\alpha/2)$

If friction is negligible
($T < 0,05P$):

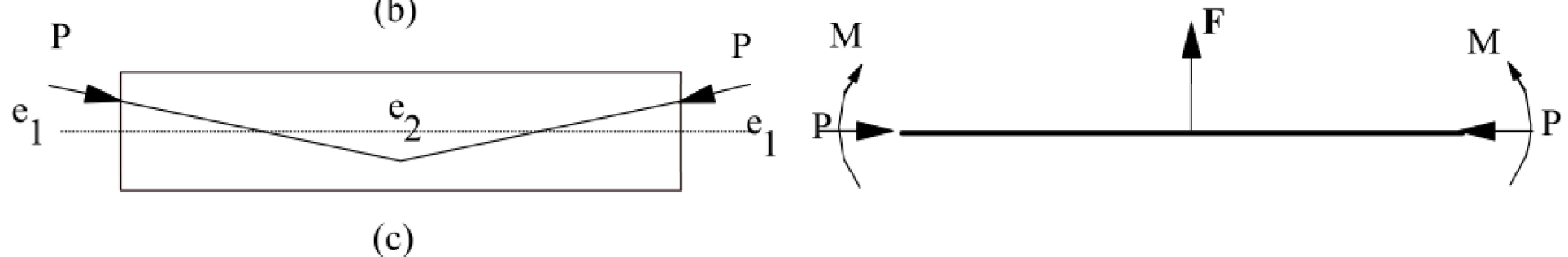
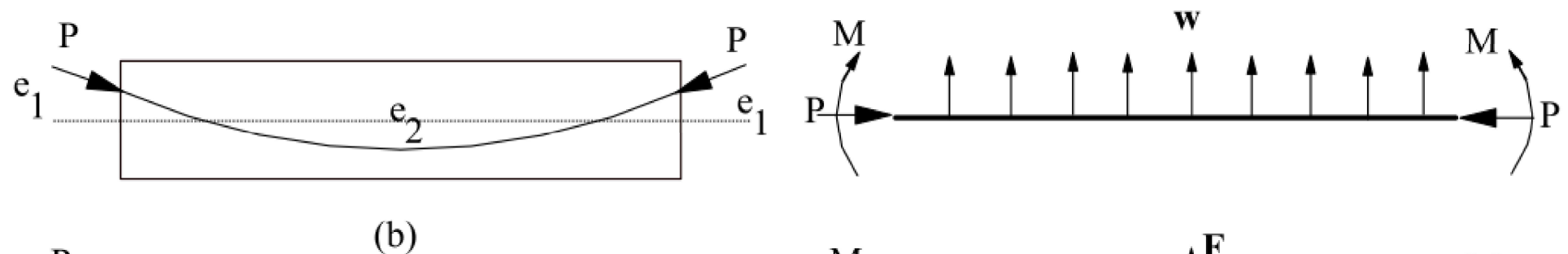
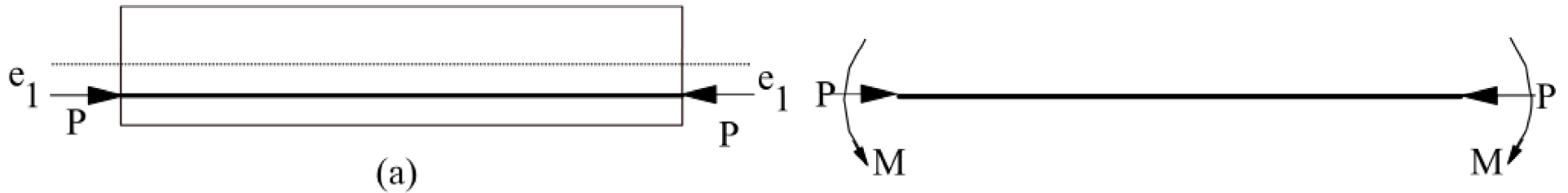
$$P_{\text{right}} = P_{\text{left}}$$

$$U = P \sin \alpha = P \alpha$$

Equivalent Load Analysis

- Another method of estimating **secondary** effects
- Can be used to calculate the total moments **directly**
- Can easily be used for **complicated profiles** and multiple spans.

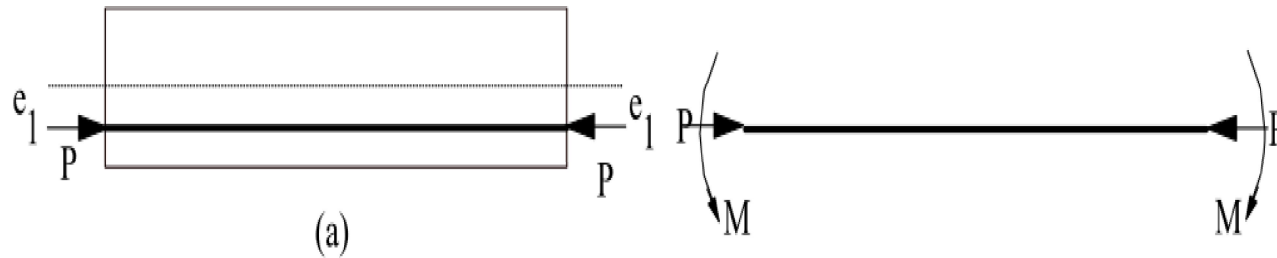
Equivalent Load Analysis



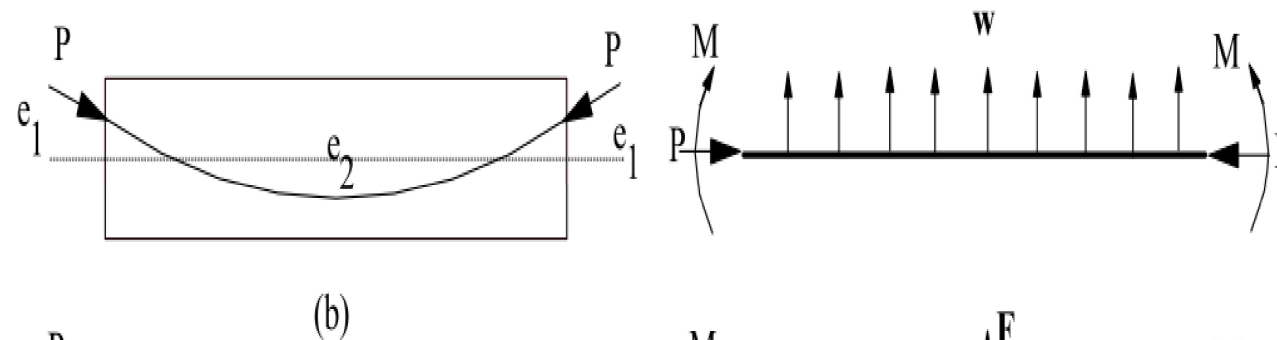
Apply these loads to the beam and you get M_{tot}

What if you want to calculate the M_{sec} ?

Equivalent Load Analysis



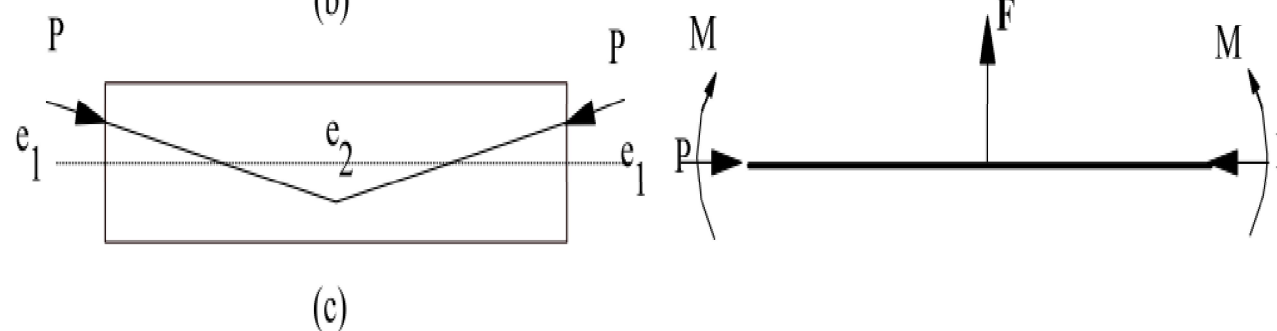
$$M = P.e_1$$



$$M = P.e_1$$

$$w = 8P(e_1 + e_2)/L^2$$

$$w = uP = P/R_{ps}$$



$$M = P.e_1$$

$$F = 4P(e_1 + e_2)/L$$